Test Cases

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Bifurcation and Model Reduction Techniques for Large Multi-Disciplinary Systems

University of Liverpool, UK, June 2008

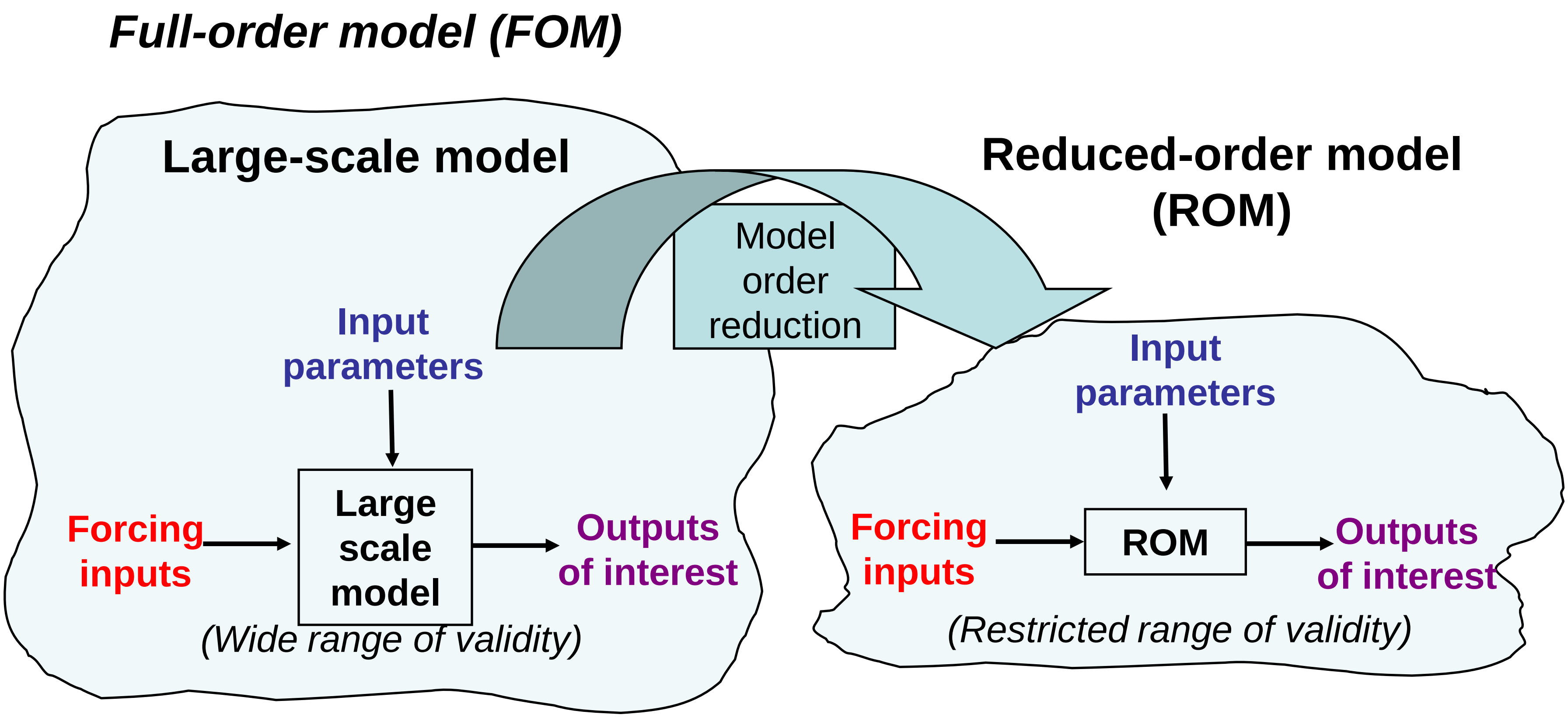
Test Cases: Basic Concept

- Limited in number
  - Multiple investigations each
- Look to capture commonalities and focus attention on weaknesses in current approaches
  - Create if not available
- Provide value to this group
  - Stimulate meaningful follow-on activity of a potentially archival nature
  - Develop generic problems that are relatively easy to interpret and communicate while being of targeted significance
  - Demonstrates shared interest
  - Think outside your discipline?

Outline

- Critique experiences at Air Force Research Laboratory from the standpoint of test cases (Day 1)
  - Model reduction techniques
  - Bifurcation and model reduction techniques
- Reflection (this evening)
- Small-group discussion (tomorrow)
- Large-group discussion (tomorrow)

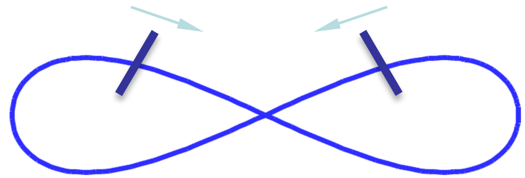
Model Reduction



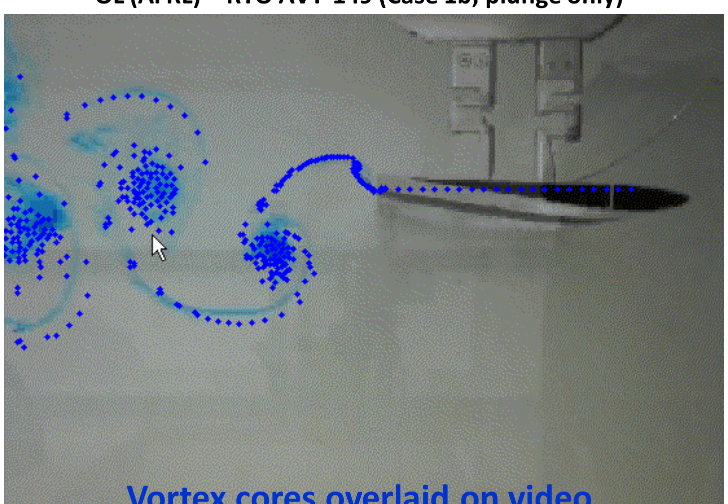
Heinkenschloss, Sorensen, Fidkowski, Willcox (2008)

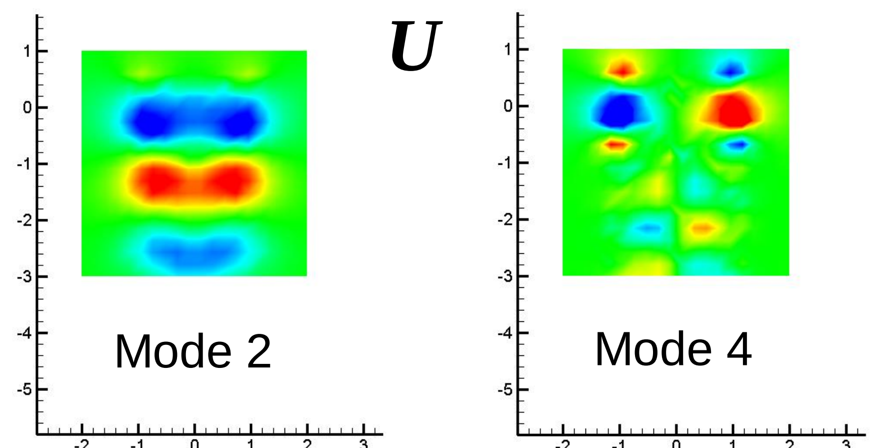
Model Reduction for Unsteady Flows

- **Objective**: Improve performance of flapping aircraft
- **Approach (planned)**: Actuation optimization using ROMs
- **Technology**: Model reduction of velocity fields computed with point vortex methods; POD for data compression and Higher Order Spectra for nonlinear modal couplings
- **Status**: Identification of quadratic couplings for POD modes
- **Emphasize time-periodicity (challenge for solvers?)**



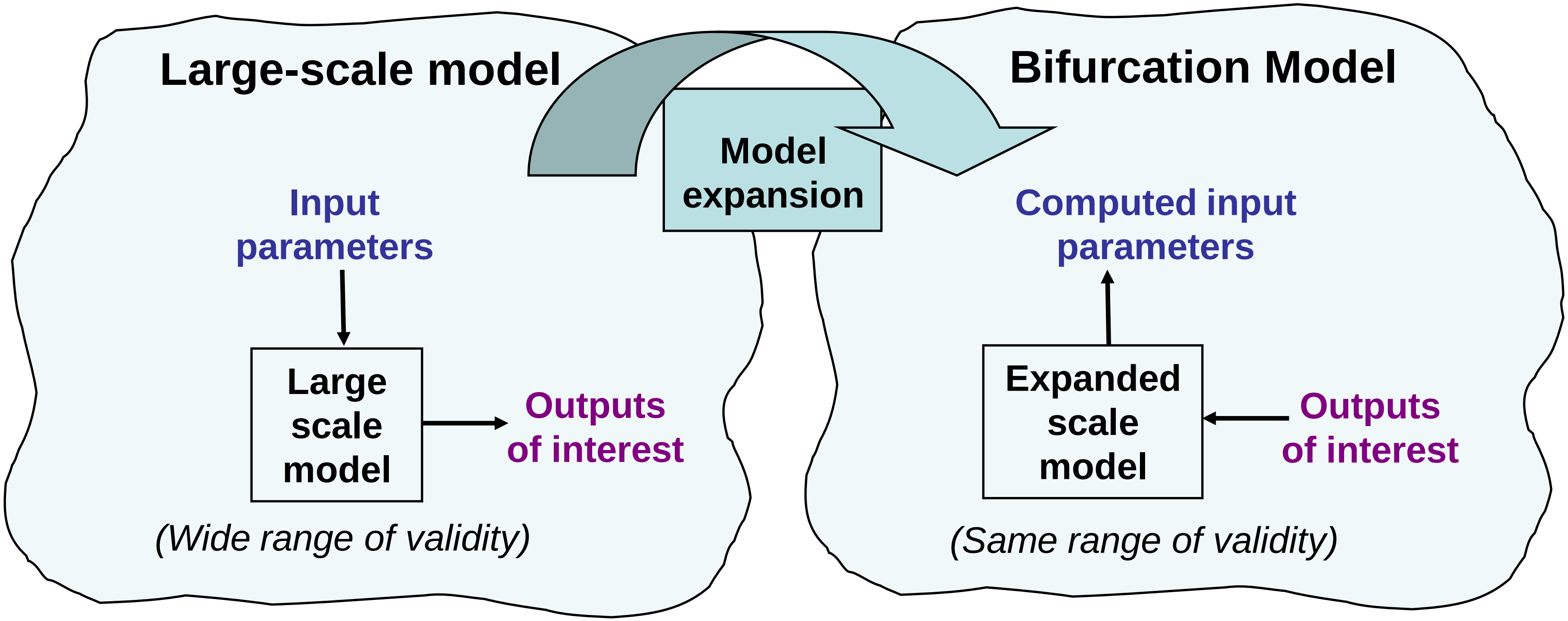
Comparison with dye injection in water channel:
OL (AFRL) – RTO AVT-149 (Case 1b; plunge only)





NATO study of airfoil motions is generic, but physics not well understood

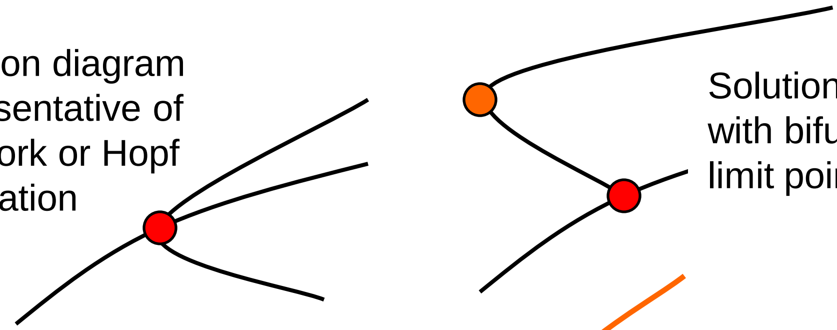
Bifurcation Techniques

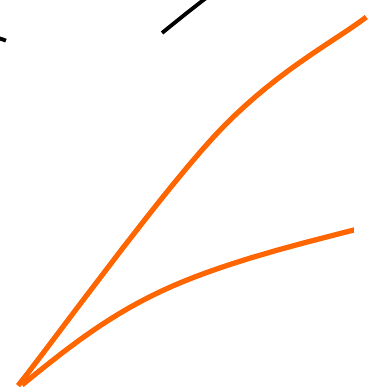


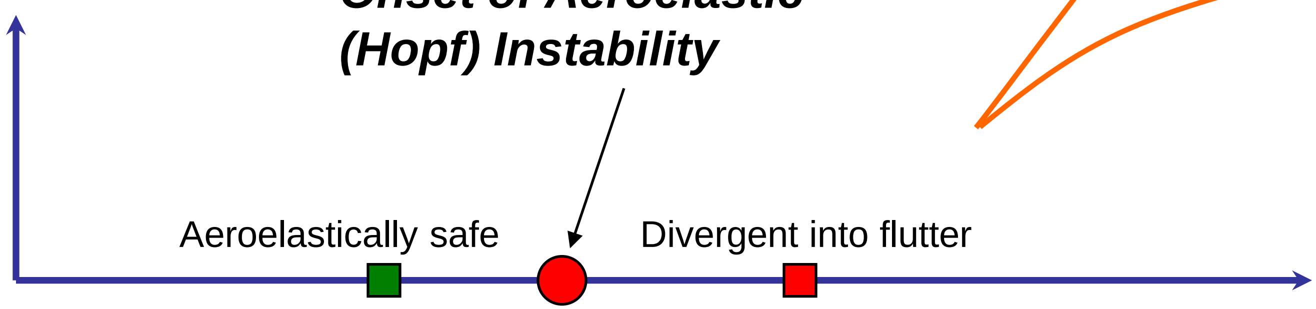
Adapted from Heinkenschloss, Sorensen, Fidkowski, Willcox (2008)

Bifurcation Techniques (cont.)

- Capture critical behaviors in large parameter spaces
  - Efficient
  - Precise
  - Ready for tracking

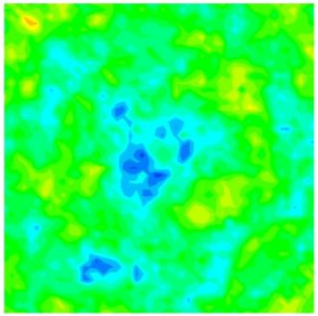






Bifurcation and Model Reduction Techniques

- **Objective**: Compute distributions of flutter speed: panel variability
- **Approach**: Geometrically nonlinear panels in supersonic flow
- **Technology**: Projection of linearized eigen-problem
- **Status**: Explored POD basis enrichment in 1-D; extending to 2-D
- **Structural plates are reasonable test case candidates**



$$\frac{d\mathbf{x}}{dt} = \mathbf{F}(\mathbf{x},\lambda) \qquad \dot{\mathbf{x}} = \mathbf{J}(\lambda)\mathbf{x} = \mathbf{L}_1\mathbf{x} + \lambda^{-1}\mathbf{L}_2\mathbf{x}$$

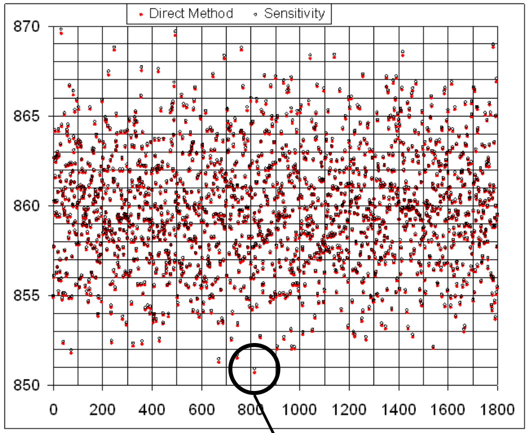
A Estimate λ^* for uniform panel with POD

$$\mathbf{z}(t) \approx \mathbf{\Phi}\hat{\mathbf{z}}(t) \qquad \frac{d\hat{\mathbf{z}}}{dt} = \mathbf{\Phi}^T\mathbf{J}_0\mathbf{\Phi}\hat{\mathbf{z}}$$

Compute eigenvalues/eigen vectors

B Solve bifurcation equations: $\Im(\mathbf{X}^*) = 0$

C Solve sensitivity equations: $\Im^*_{\mathbf{X}}\Delta\mathbf{X} = -\Im$ with variations



Bifurcation analysis = O(1 hour)
Sensitivity analysis = O(1 sec)